The University of Texas at Austin
Aerospace Engineering
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Cockrell School of Engineering

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MICROSTRUCTURE-BASED ESTIMATION OF THE EFFECTIVE STIFFNESS OF CROSSLINKED, EMBEDDED FIBER NETWORKS

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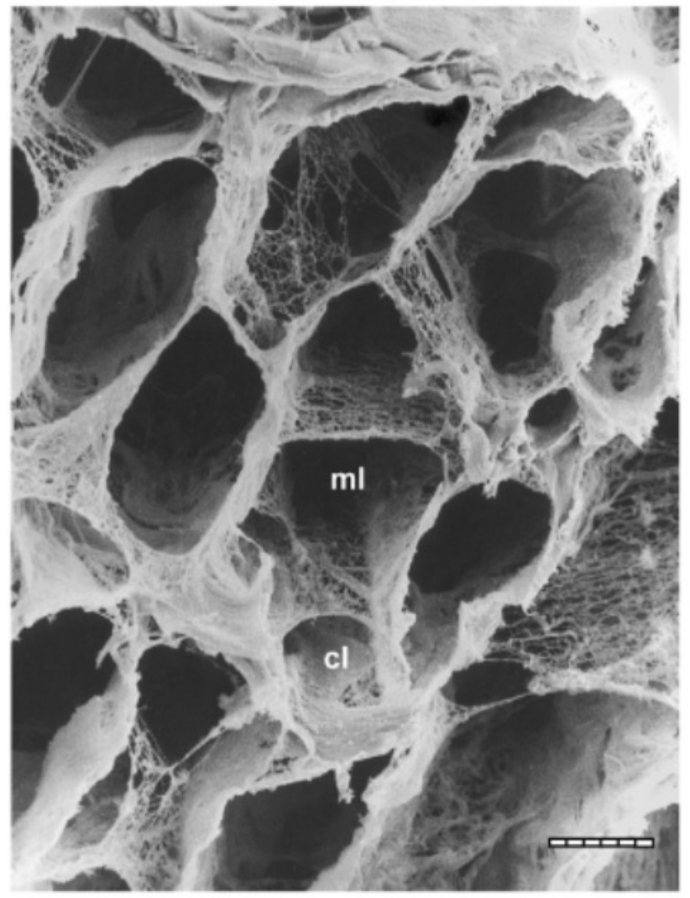
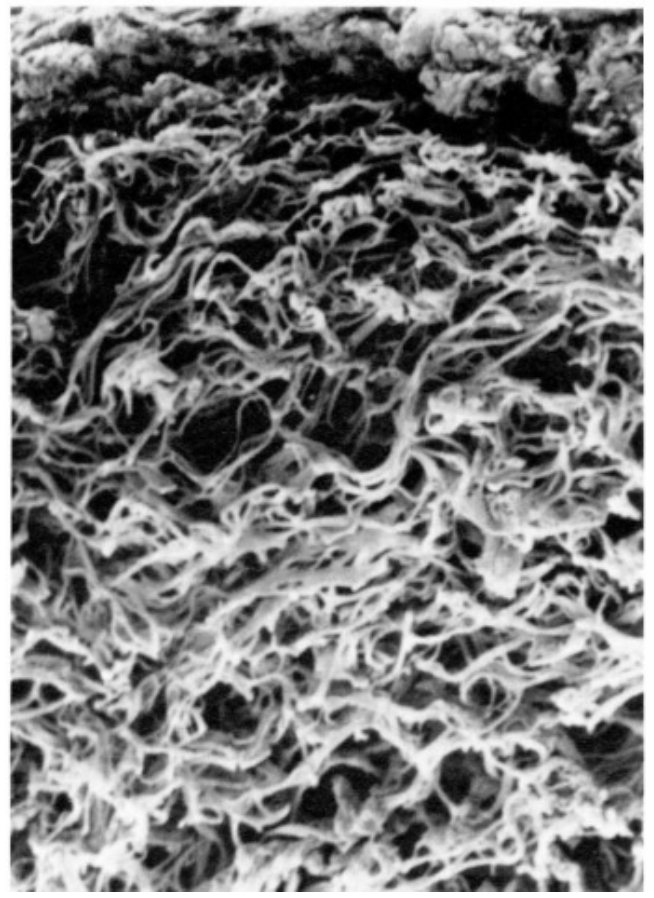
Introduction

- Semi-flexible biopolymers are ubiquitous building blocks of life, often organized in **fibrous networks**
  - Collagen networks in myocardium, skin, blood vessels, ligaments, tendons etc.
  - Fibrin networks in blood clots

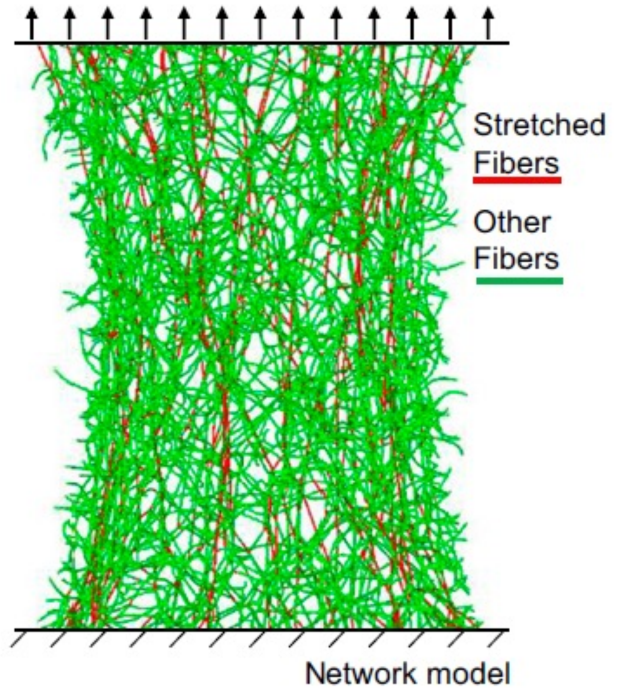
B. Intrigila, et al. (2007)



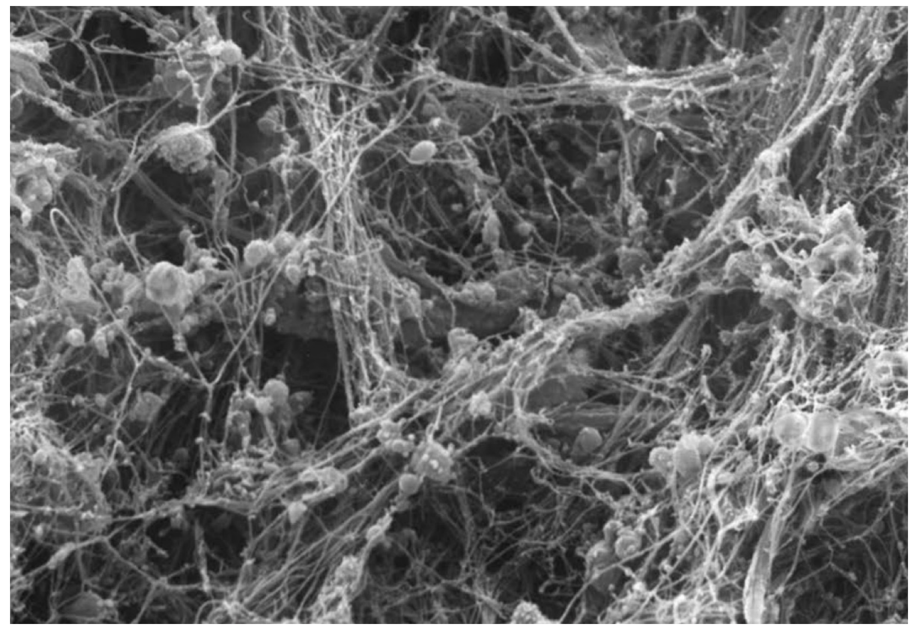
L.T. Smith, et al. (1982)



- They exhibit **complex mechanical phenomena**
  - Strong nonlinearities
  - Strain stiffening
  - Anomalous Poisson's effect
  - Negative Poynting effect

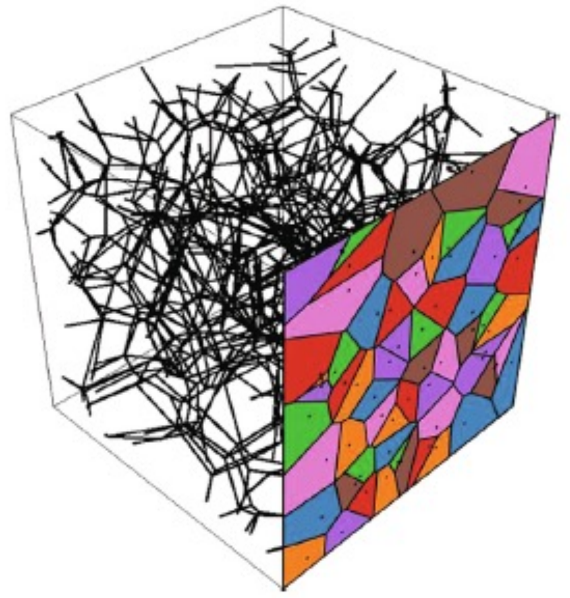


Ban et al. (2019)



Chernysh, Irina N., et al. *Scientific reports* 10.1 (2020)

- Previous efforts have modeled fiber networks in **isolation:**
  - No embedding matrix
  - Elastic behavior
  - Discrete elements



Merson & Picu (2020)

- Why in isolation? – **Discretization**

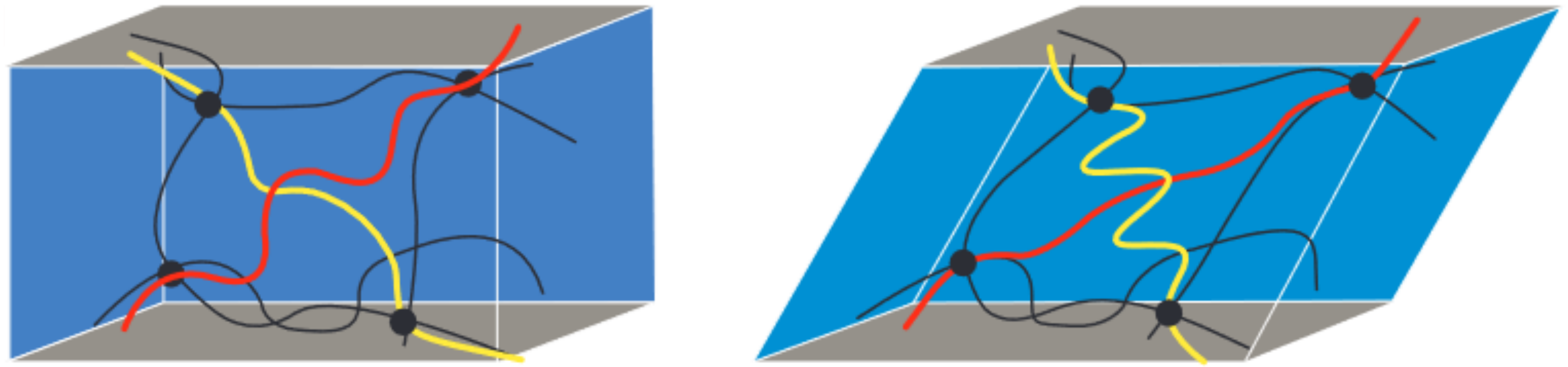


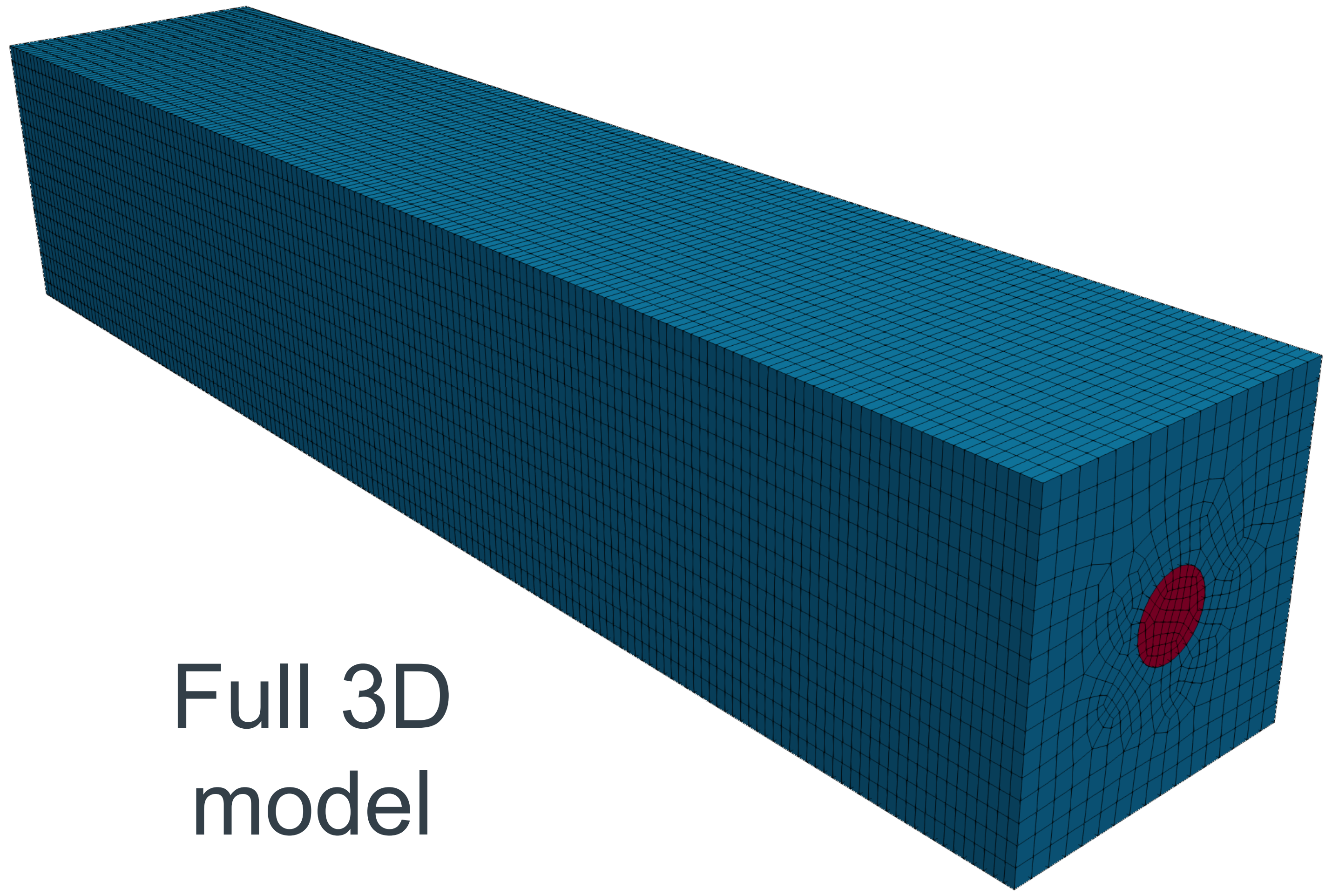
Figure adapted from Janmey P. A., et al. (2016)

Introduction

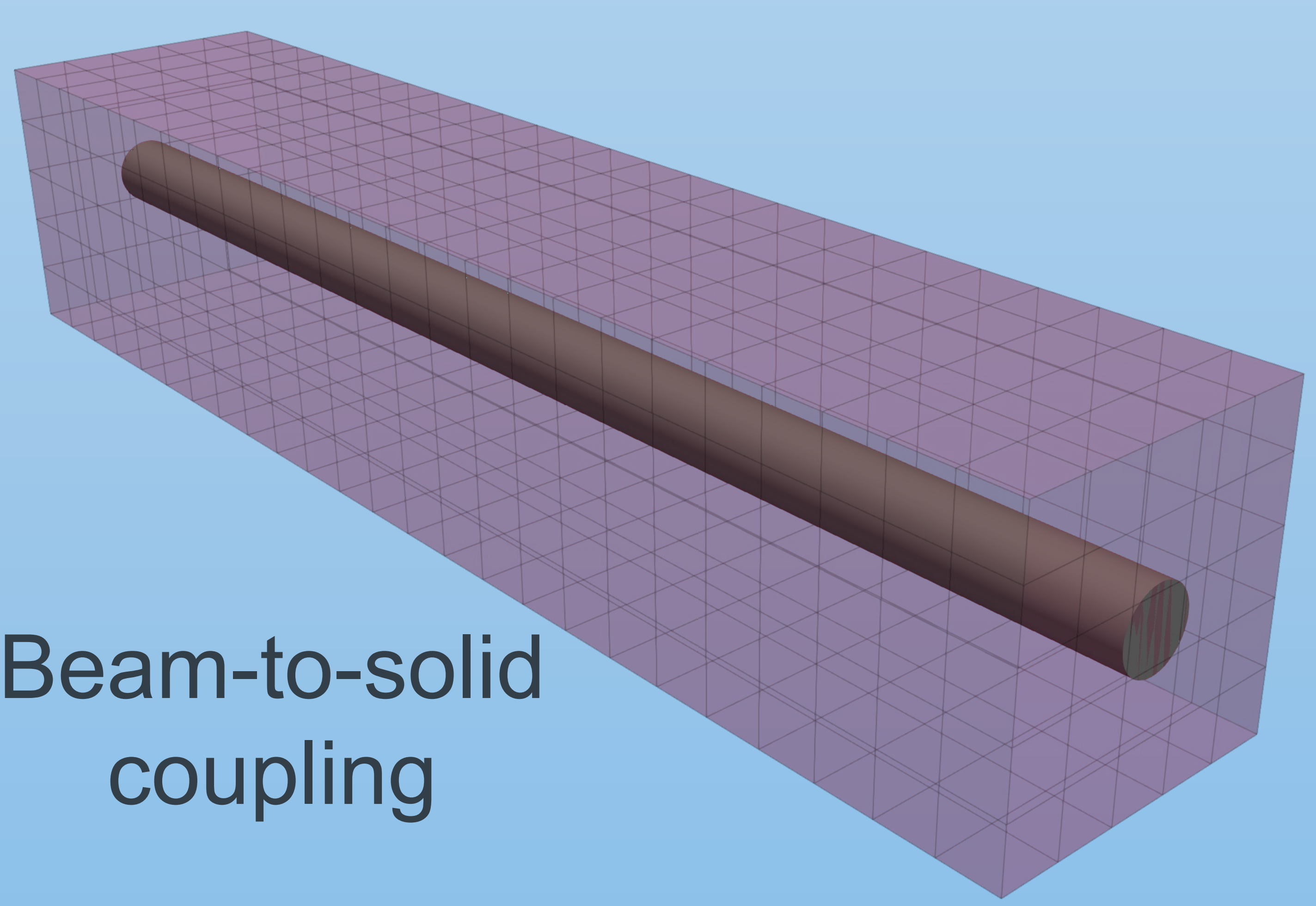
- **Motivation**: Delineate contributions of each constituent: **matrix** and **fibers**
  - How does network architecture affect the mechanics/ apparent stiffness?
  - Mean fiber length?
  - Fiber undulations?

- **Objective**: Develop a computationally efficient model of the elastic behavior of embedded fiber networks under large deformation.

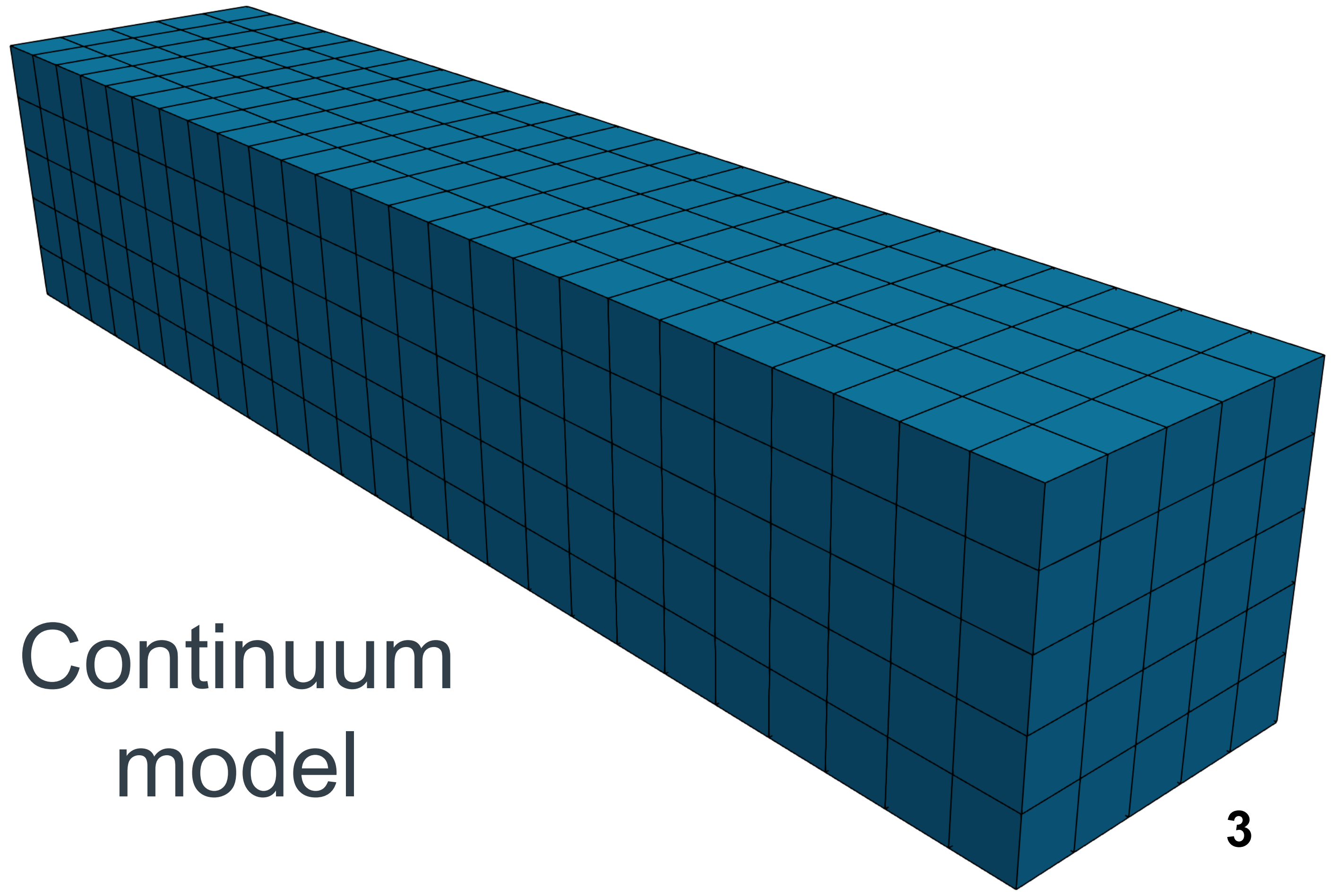
- Modeling approaches for embedded elastic fibers:



Full 3D model

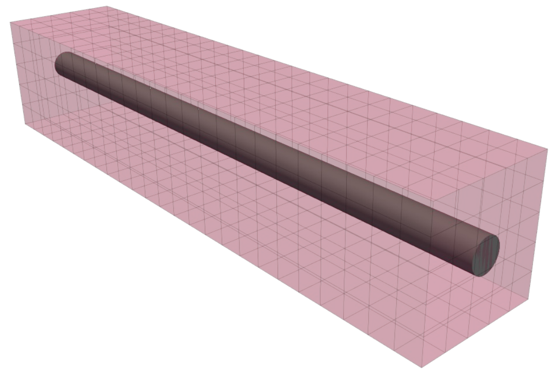


Beam-to-solid coupling



Continuum model

Theory



- Based on previous work by Steinbrecher, Ivo, et al. "**A mortar-type finite element approach for embedding 1D beams into 3D solid volumes.**" *Computational Mechanics* (2020).

- Coupling Constraint:

$$\underline{\boldsymbol{u}}^B - \underline{\boldsymbol{u}}^S = \underline{\boldsymbol{0}} \text{ on } \Gamma_c^{1D-3D} = \Omega^B \text{ (beam centerline)}$$

- Principle of virtual work:

Solid Beam Coupling

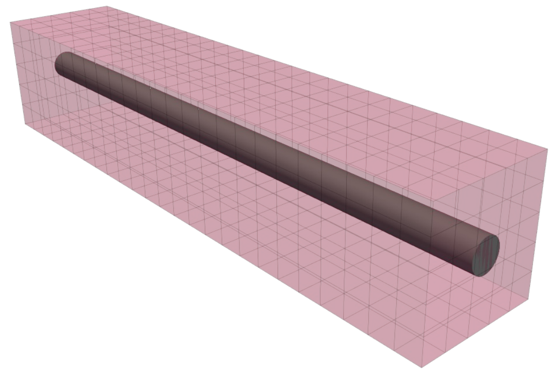
$$\delta W^S + \delta W^B + \delta W^C = 0$$

$$\delta W^C = -\delta W_c + \delta W_\lambda = \int_{\Gamma_c^{1D-3D}} \underline{\boldsymbol{\lambda}}(\delta \underline{\boldsymbol{u}}^B - \delta \underline{\boldsymbol{u}}^S) ds + \int_{\Gamma_c^{1D-3D}} \delta \underline{\boldsymbol{\lambda}}(\underline{\boldsymbol{u}}^B - \underline{\boldsymbol{u}}^S) ds$$

where

$\underline{\boldsymbol{\lambda}} \in \mathbb{R}^3$: Lagrange multiplier field (interface line load)

Theory



- Linearized system:

$$\begin{bmatrix} \boldsymbol{K}_{SS} & \mathbf{0} & -\boldsymbol{M}^T \\ \mathbf{0} & \boldsymbol{K}_{BB} & \boldsymbol{D}^T \\ -\boldsymbol{M} & \boldsymbol{D} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{d}^{\mathrm{S}} \\ \Delta\mathbf{d}^{\mathrm{B}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{f}_{int}^{\mathrm{S}} + \boldsymbol{f}_{ext}^{\mathrm{S}} \\ -\boldsymbol{f}_{int}^{\mathrm{B}} + \boldsymbol{f}_{ext}^{\mathrm{B}} \\ \boldsymbol{g}_c \end{bmatrix} \quad \text{where} \quad \boldsymbol{g}_c(\mathbf{d}^{\mathrm{S}}, \mathbf{d}^{\mathrm{B}}) = \begin{bmatrix} -\boldsymbol{M} & \boldsymbol{D} \end{bmatrix} \begin{bmatrix} \mathbf{d}^{\mathrm{S}} \\ \mathbf{d}^{\mathrm{B}} \end{bmatrix}$$

- Enforce coupling constraint using the penalty method and setting $\boldsymbol{\lambda} = \varepsilon \boldsymbol{\kappa}^{-1} \boldsymbol{g}_c(\mathbf{d}^{\mathrm{S}}, \mathbf{d}^{\mathrm{B}})$

$$\begin{bmatrix} \boldsymbol{K}_{SS} + \varepsilon \boldsymbol{M}^T \boldsymbol{\kappa}^{-1} \boldsymbol{M} & -\varepsilon \boldsymbol{M}^T \boldsymbol{\kappa}^{-1} \boldsymbol{D} \\ -\varepsilon \boldsymbol{D}^T \boldsymbol{\kappa}^{-1} \boldsymbol{M} & \boldsymbol{K}_{BB} + \varepsilon \boldsymbol{D}^T \boldsymbol{\kappa}^{-1} \boldsymbol{D} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{d}^{\mathrm{S}} \\ \Delta\mathbf{d}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{f}_{int}^{\mathrm{S}} + \boldsymbol{f}_{ext}^{\mathrm{S}} - \boldsymbol{f}_{c}^{\mathrm{S}} \\ -\boldsymbol{f}_{int}^{\mathrm{B}} + \boldsymbol{f}_{ext}^{\mathrm{B}} - \boldsymbol{f}_{c}^{\mathrm{B}} \end{bmatrix}$$

with

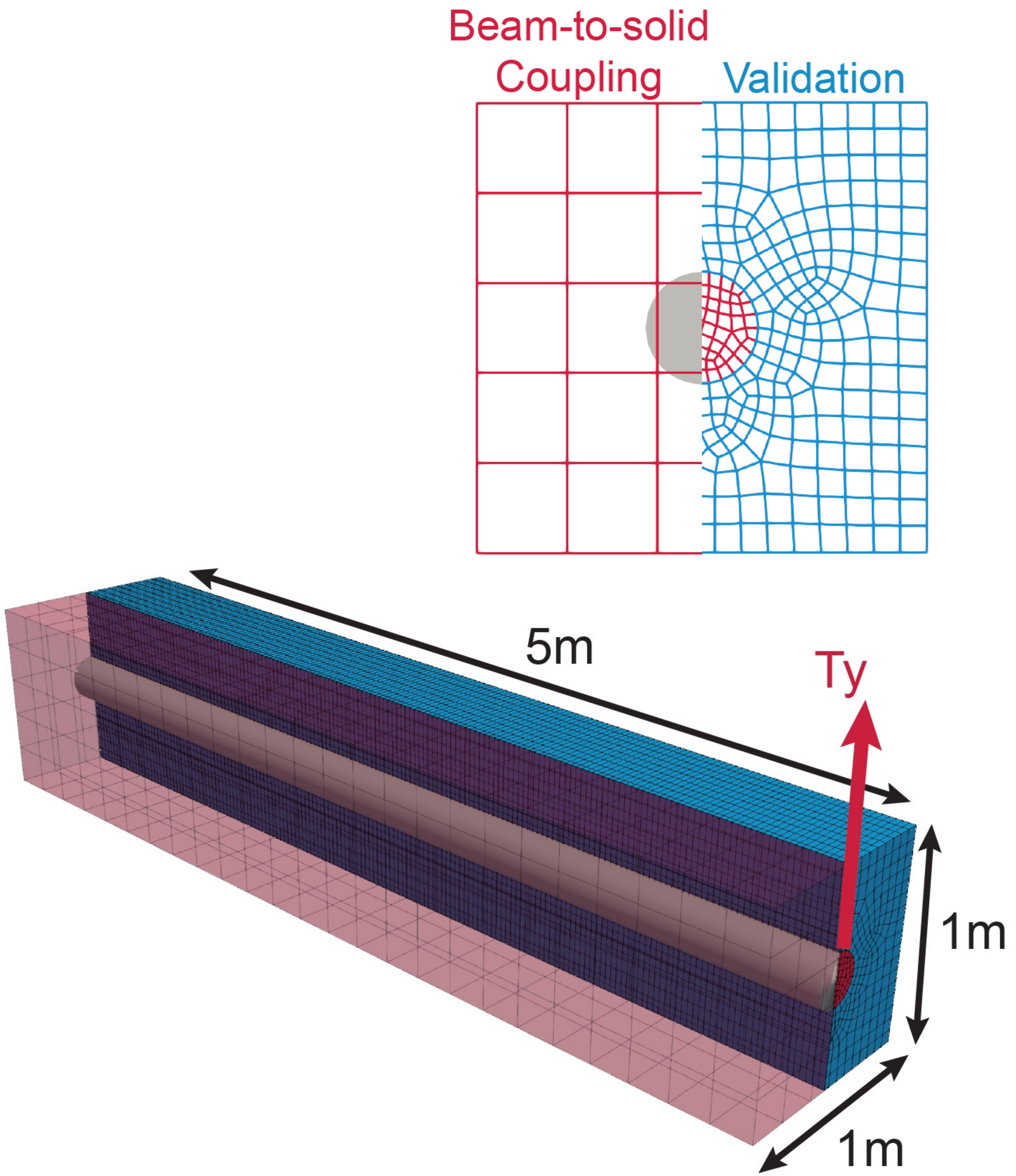
$$-\boldsymbol{f}_c^{\mathrm{S}} = \varepsilon \boldsymbol{M}^T \boldsymbol{\kappa}^{-1} \begin{bmatrix} -\boldsymbol{M} & \boldsymbol{D} \end{bmatrix} \begin{bmatrix} \mathbf{d}^{\mathrm{S}} \\ \mathbf{d}^{\mathrm{B}} \end{bmatrix}$$

$$-\boldsymbol{f}_c^{\mathrm{B}} = \varepsilon \boldsymbol{D}^T \boldsymbol{\kappa}^{-1} \begin{bmatrix} -\boldsymbol{M} & \boldsymbol{D} \end{bmatrix} \begin{bmatrix} \mathbf{d}^{\mathrm{S}} \\ \mathbf{d}^{\mathrm{B}} \end{bmatrix}$$

Validation

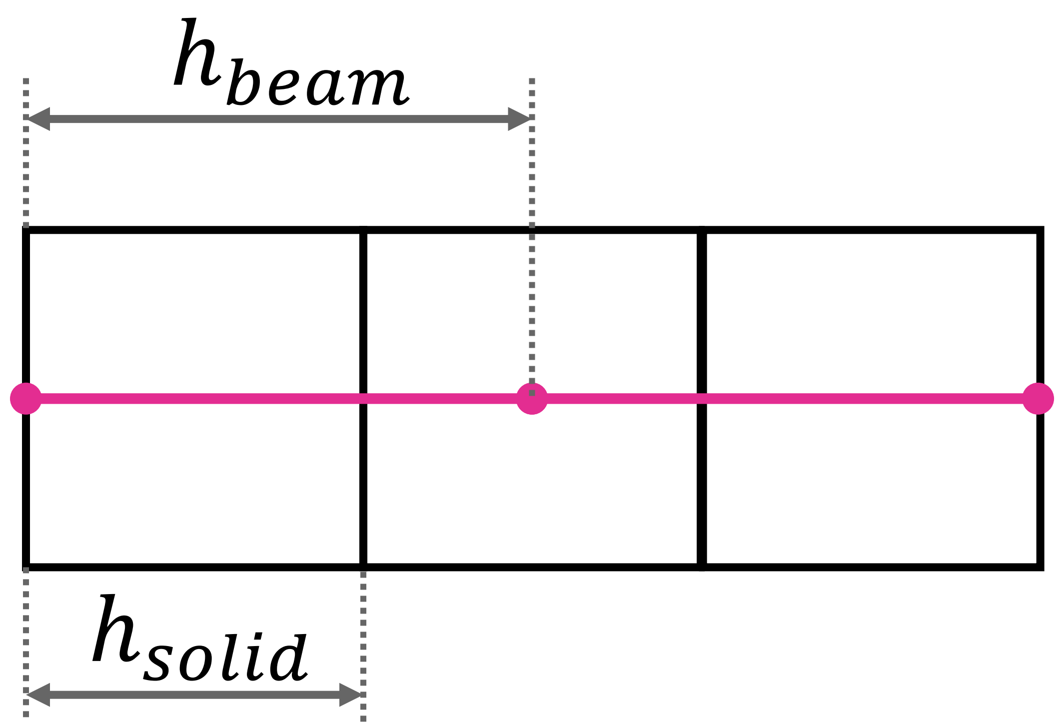
- Reinforced cantilever beam, fixed on the left end, applied distributed load on the free face (right end).
- Comparison between the full 3D model (reference solution) and our beam-to-solid coupling Abaqus implementation.
- Displacement error of the solid domain:

$$\|e\| = \sqrt{\frac{\int_{V_0} \left\|\underline{\boldsymbol{u}}^S - \underline{\boldsymbol{u}}^S_{ref}\right\|^2 dV_0}{\int_{V_0} \left\|\underline{\boldsymbol{u}}^S_{ref}\right\|^2 dV_0}}$$



Validation – Sensitivity Studies

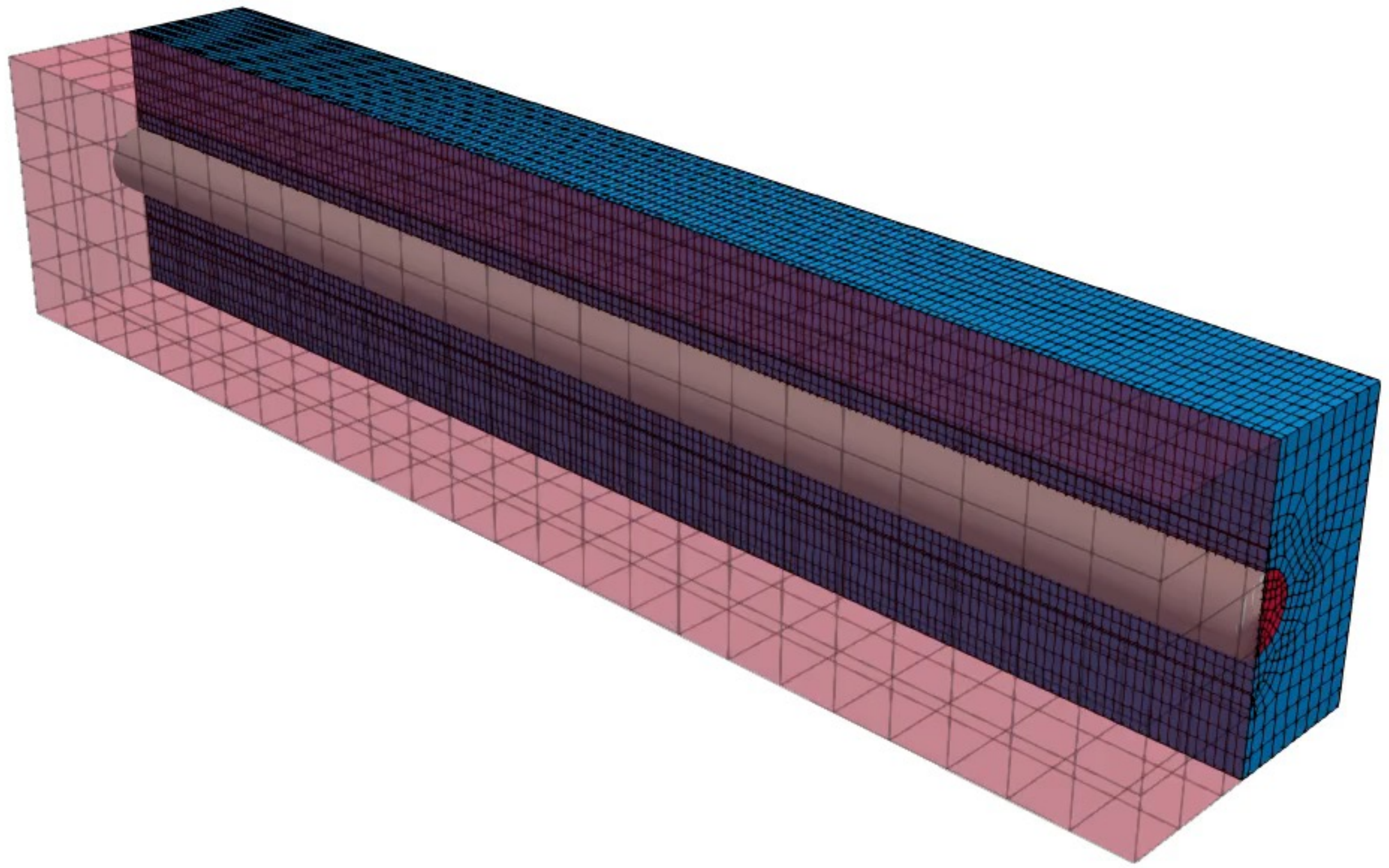
- Limitations
  - Beam element size > Solid element size
  - Solid element size ≈ Fiber radius

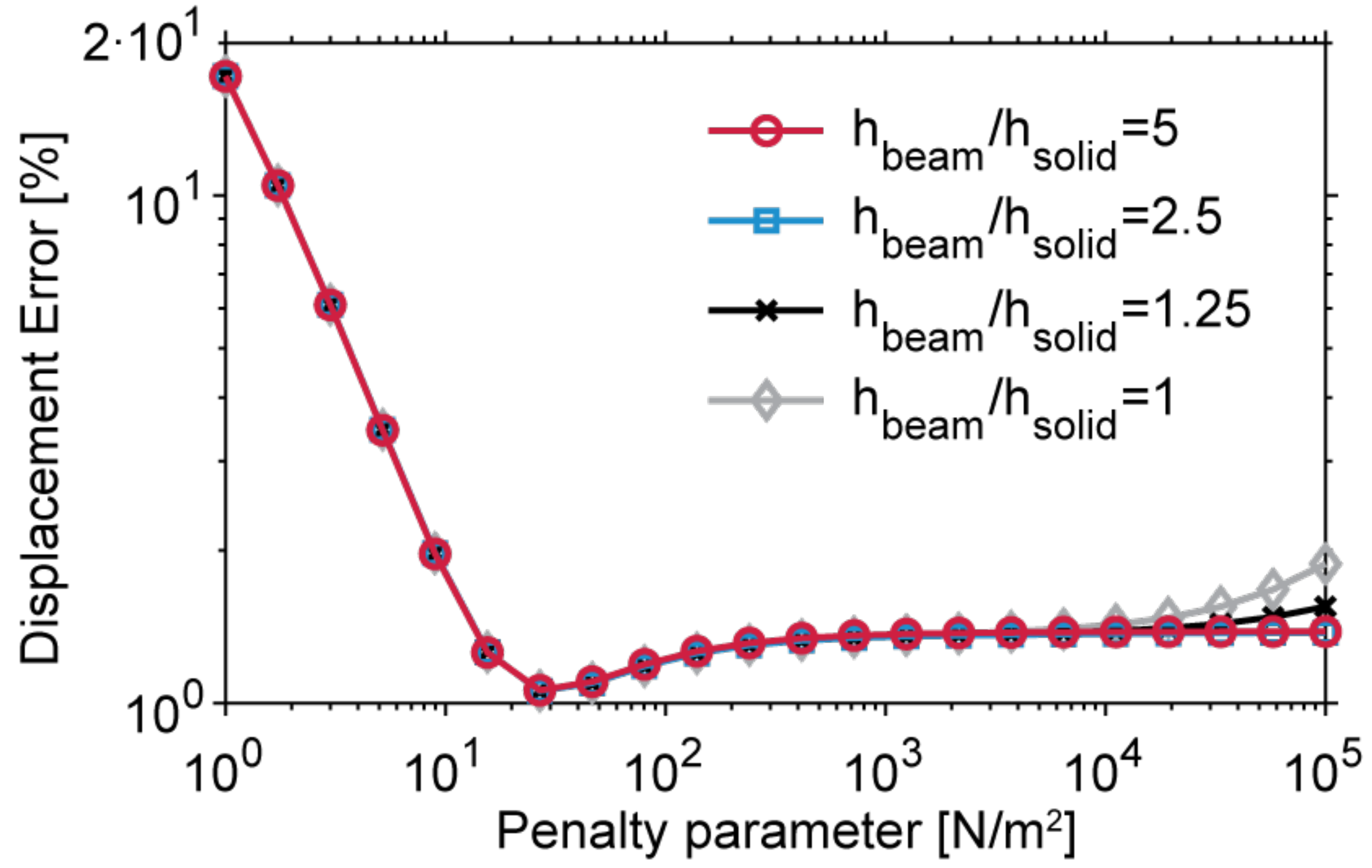


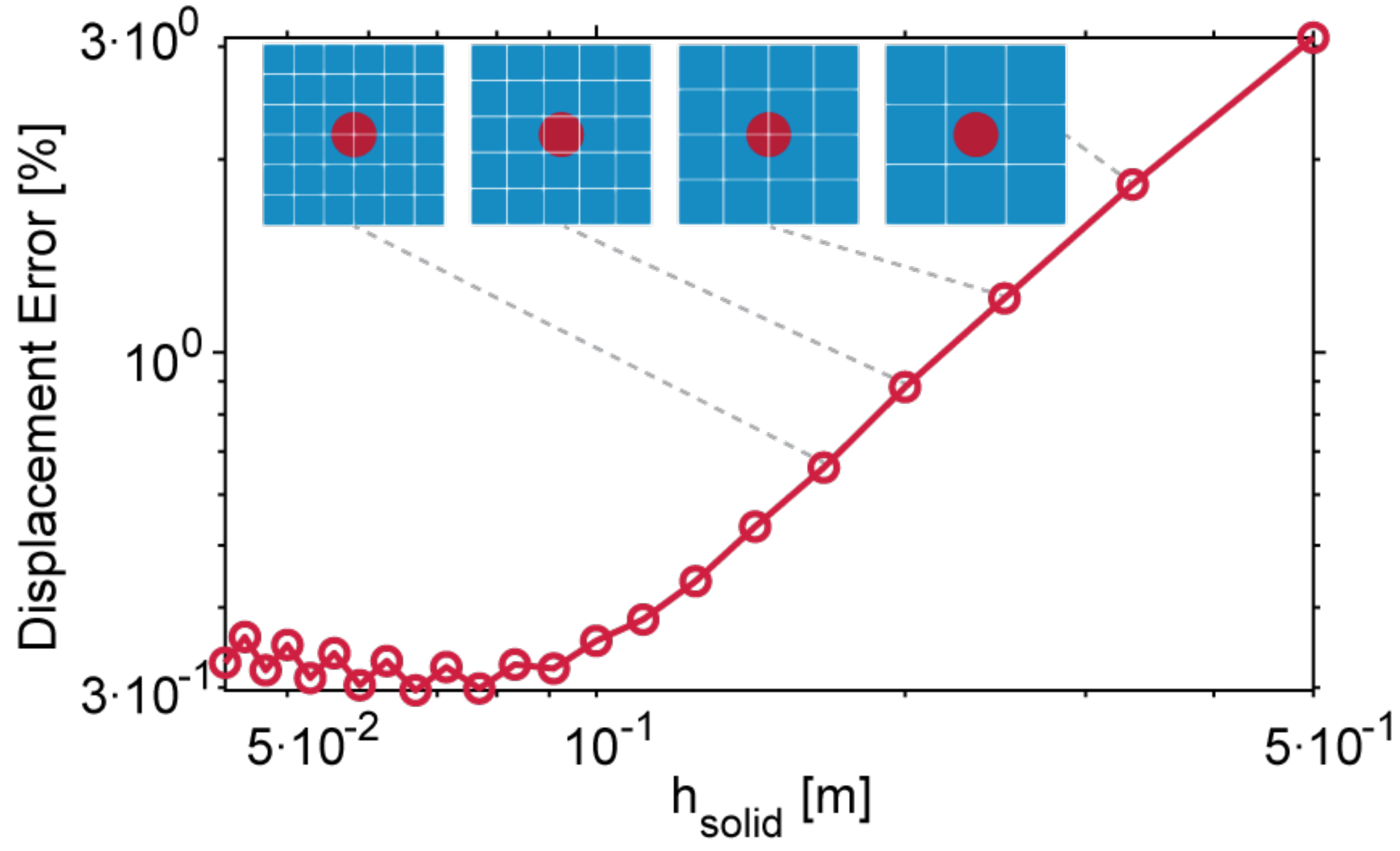
- Displacement error: < 1.0%

- Reference solution
  - Solid elements: 75,985
  - CPU time: 2210 sec

- Beam-to-solid coupling
  - Solid elements: 625
  - CPU time: 29 sec

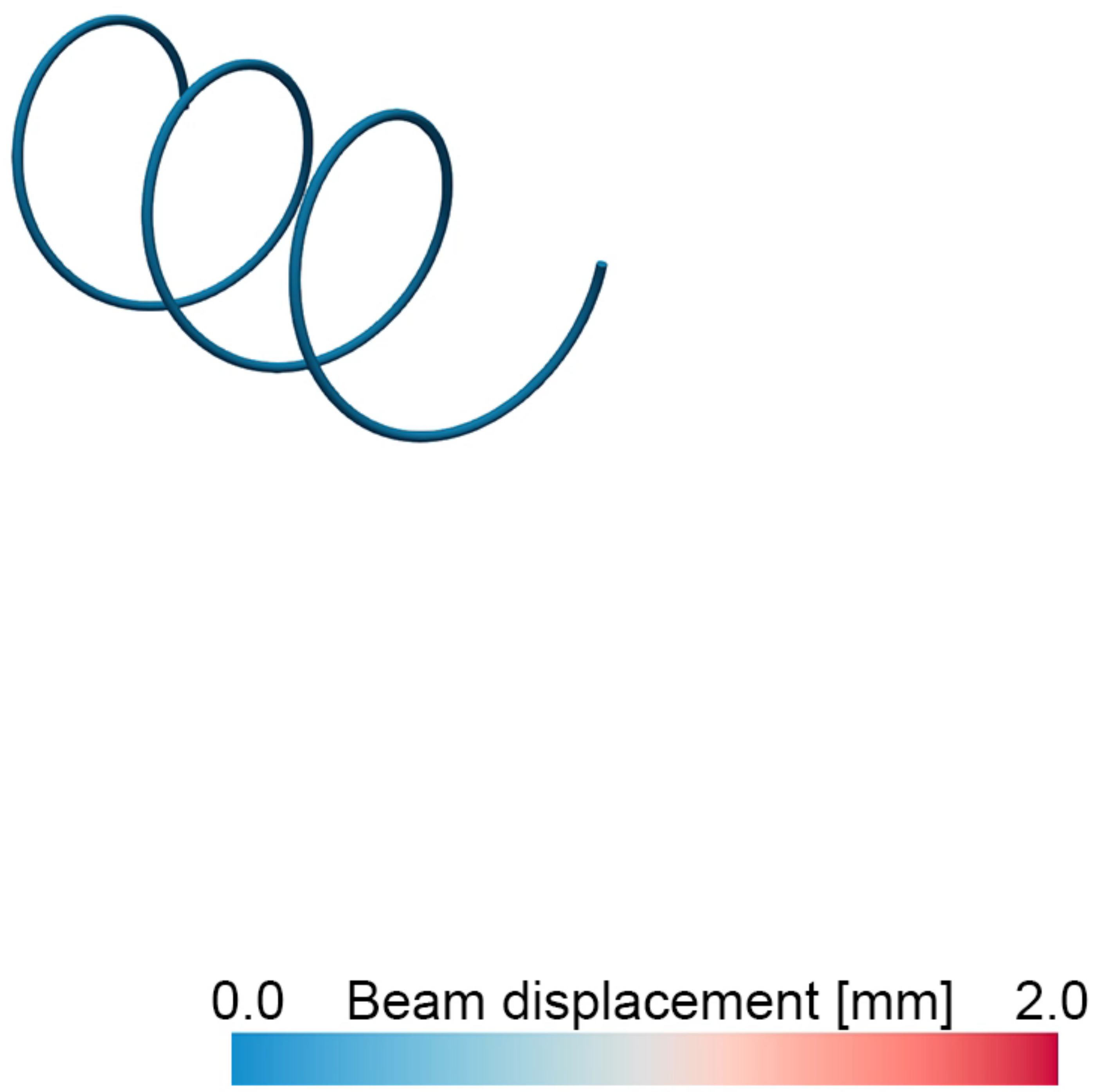


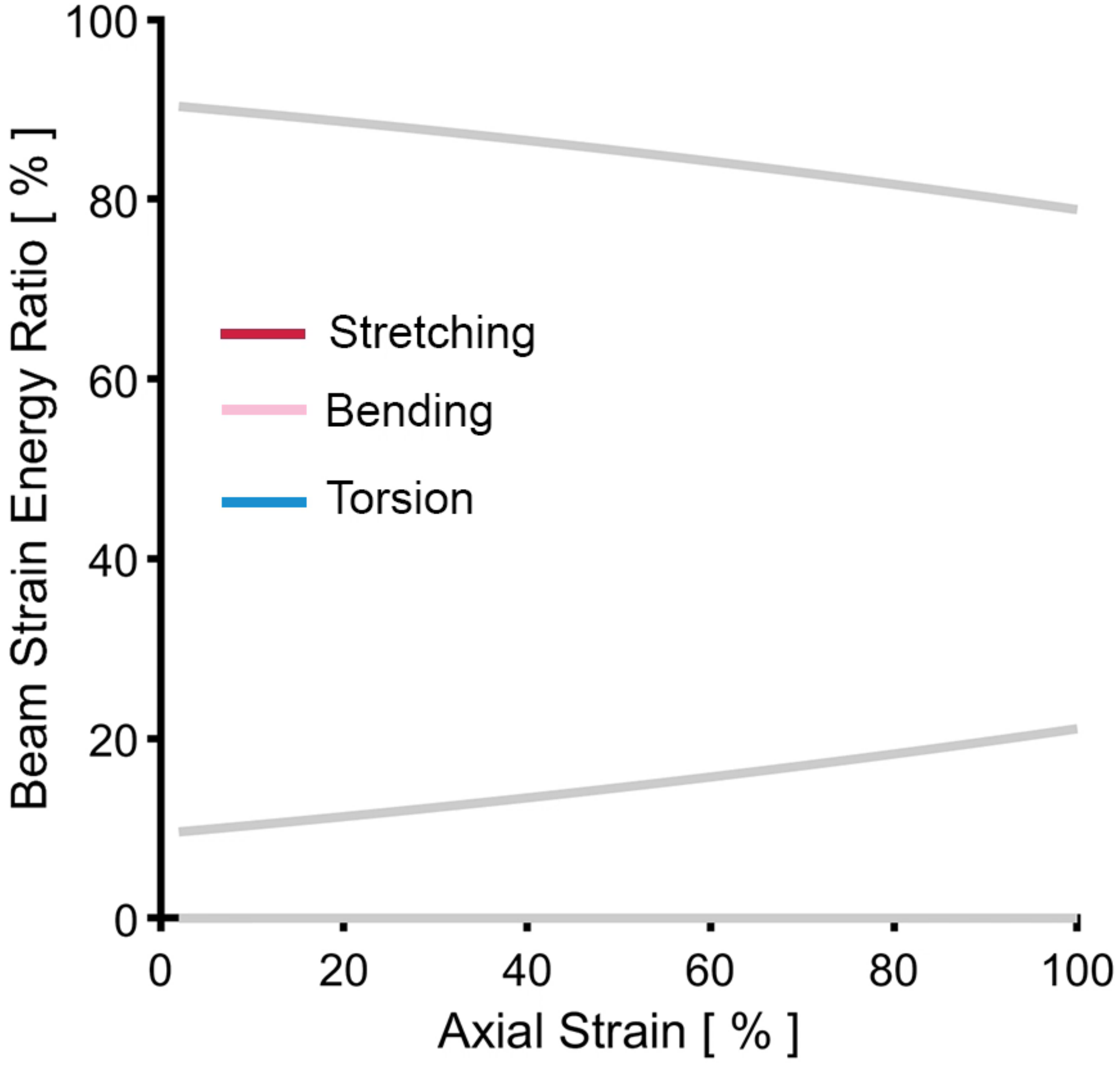




Example: Helical Beam

- Spatial Timoshenko beam.
- Linear elastic material law.
- Uniaxial extension to 100% strain.
- Strain energy components:
  - Axial stretching
  - Bending
  - Torsional

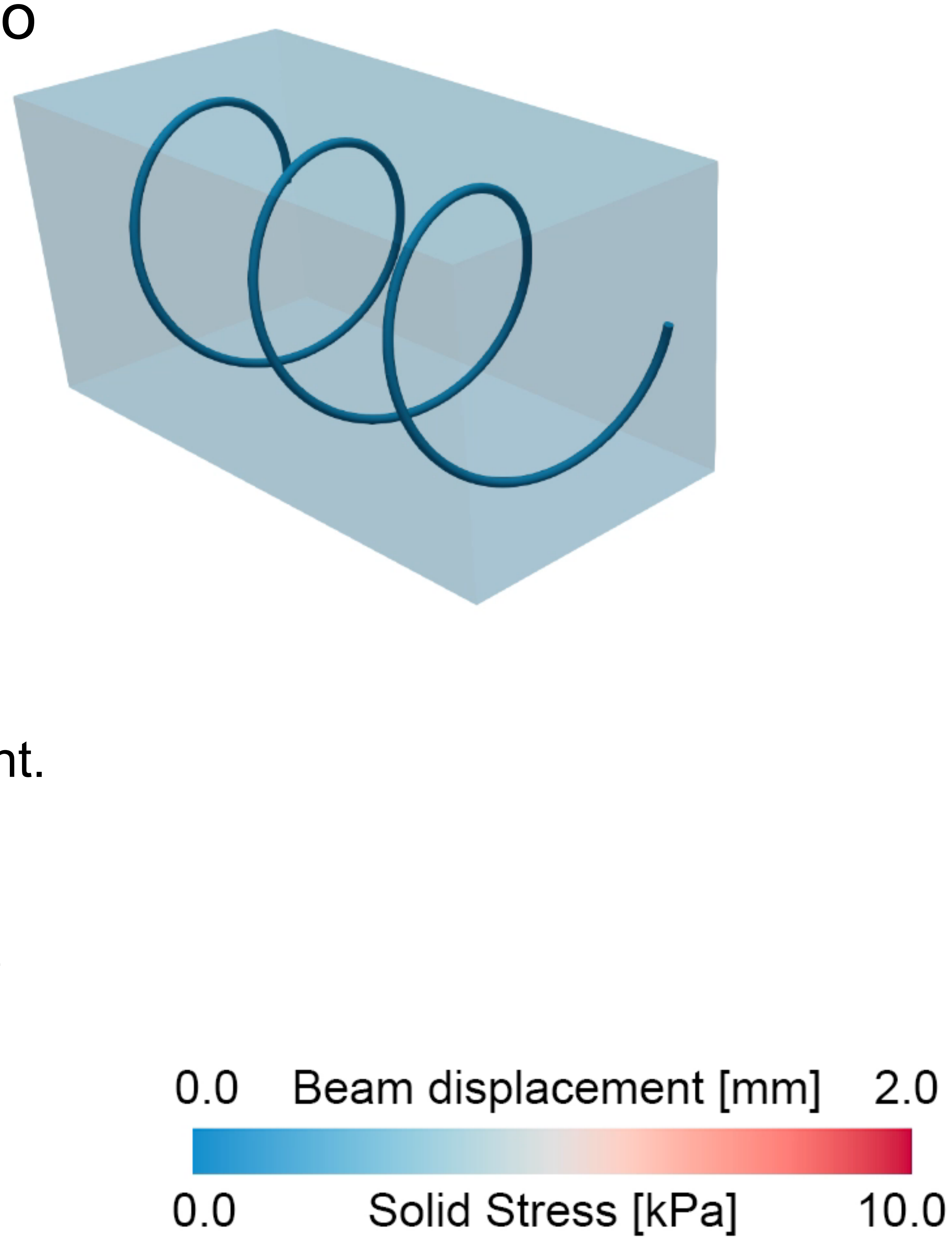


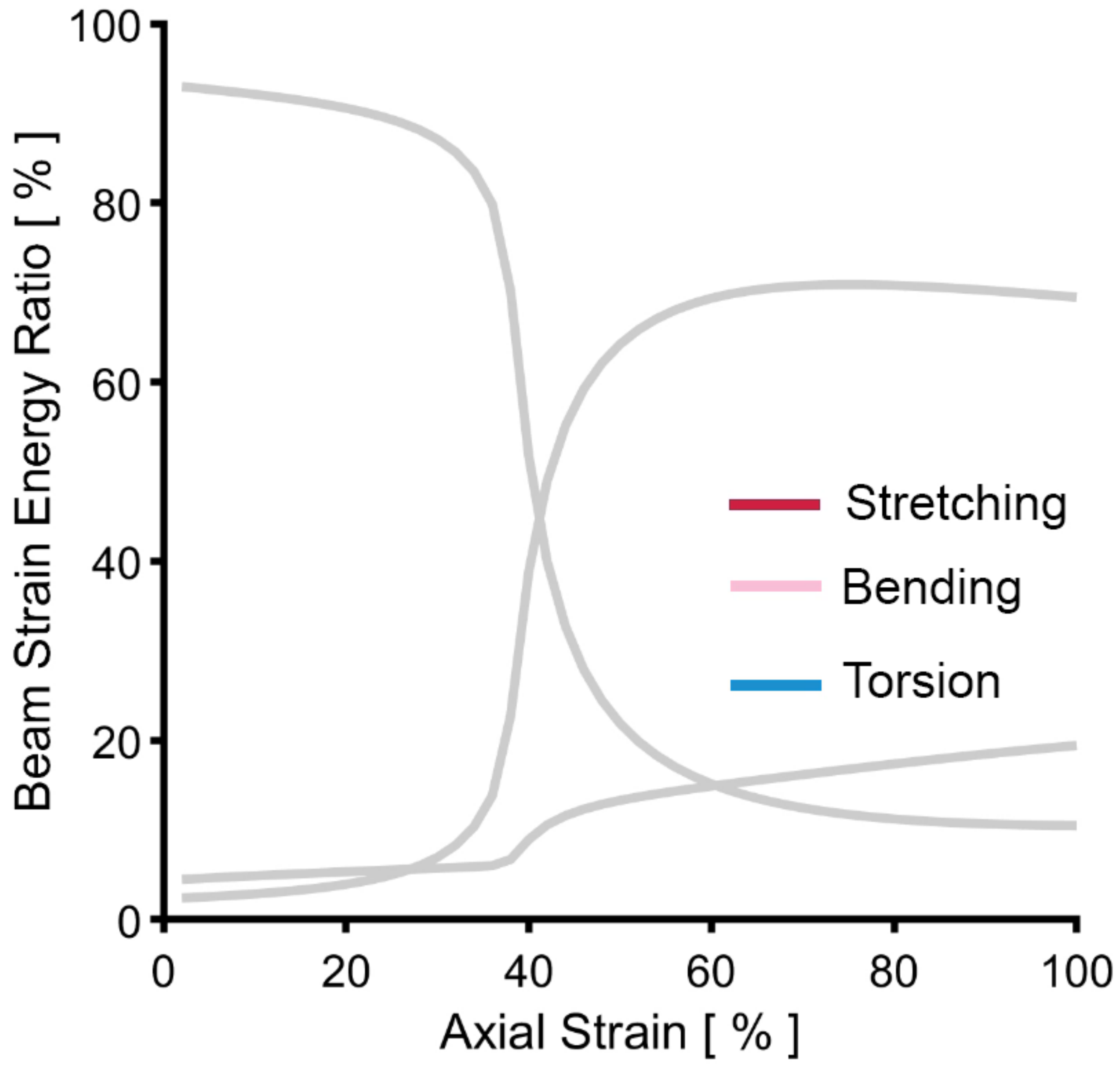


Example: Helical Beam

- Same beam, embedded into isotropic, incompressible Neo-hookean material.

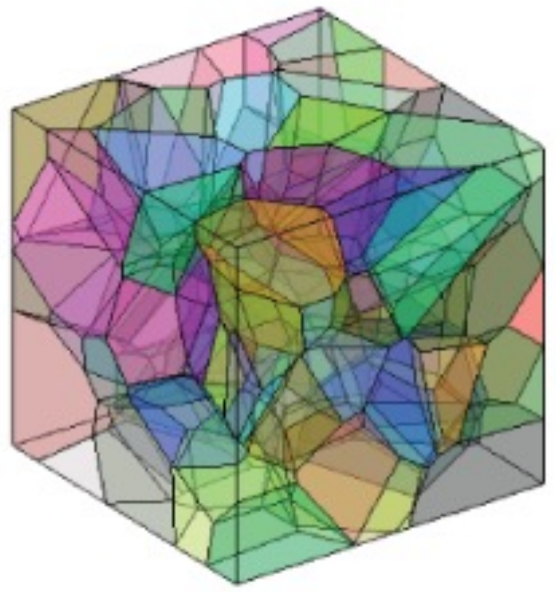
- Our model is able to:
  - Capture beam instabilities caused by the solid-to-beam interaction forces.
  - Delineate the contribution of each strain energy component.
  - Investigate the effect of the relative stiffness between the solid matrix and the beam.





Embedded Fiber Networks

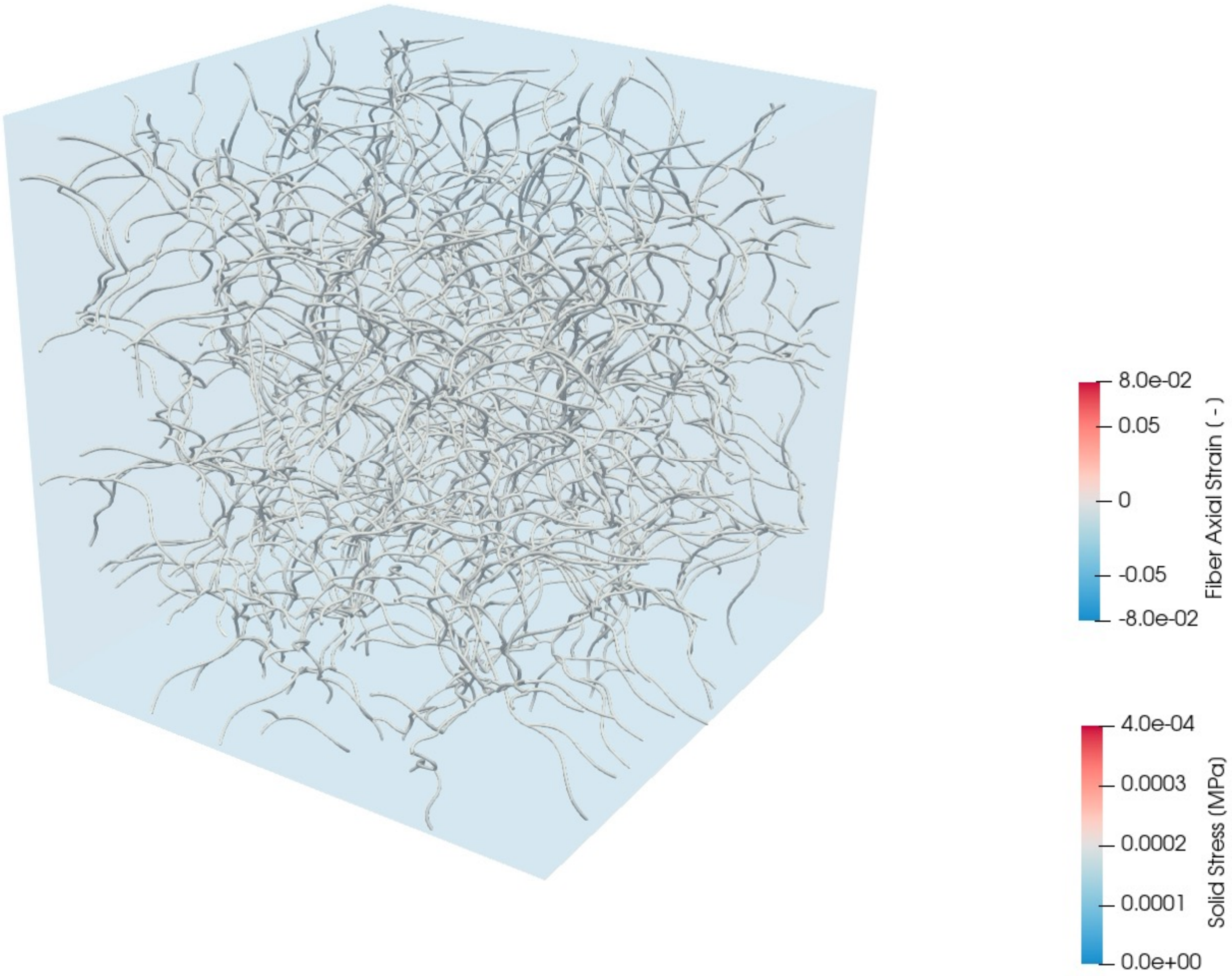
- Voronoi-based networks
  - Average connectivity number <z>=3.4
  - Introduce sinusoidal undulations



- Simple shear deformation
  - Rigid displacement boundary conditions
  - Cubic geometry
  - Deformed up to 50% shear strain

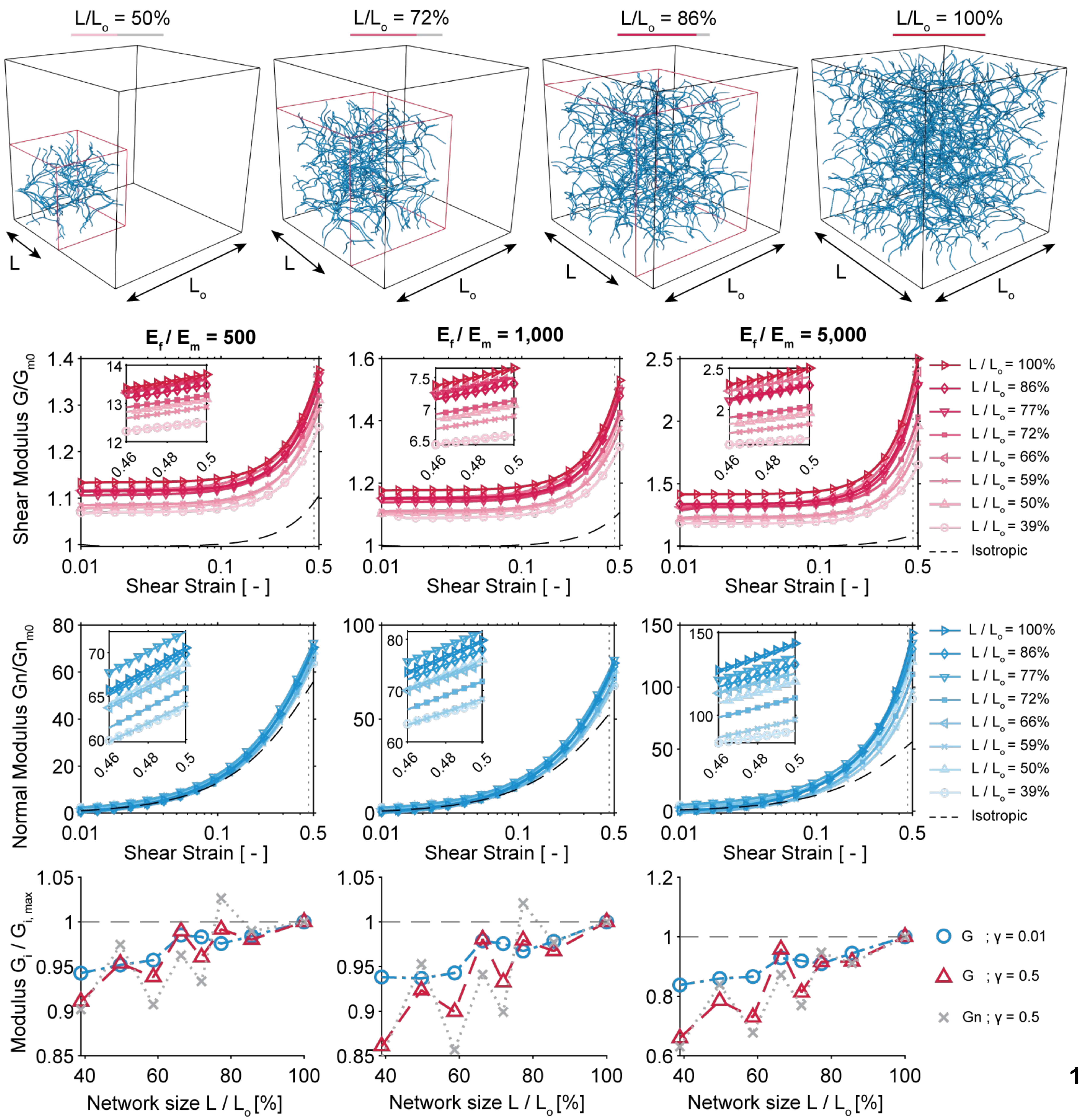
- Effective Stiffness
  - Shear & Normal moduli

$$G = \frac{\Delta\sigma_{xy}}{\Delta\gamma}, \quad G_n = \frac{\Delta\sigma_{yy}}{\Delta\gamma}$$

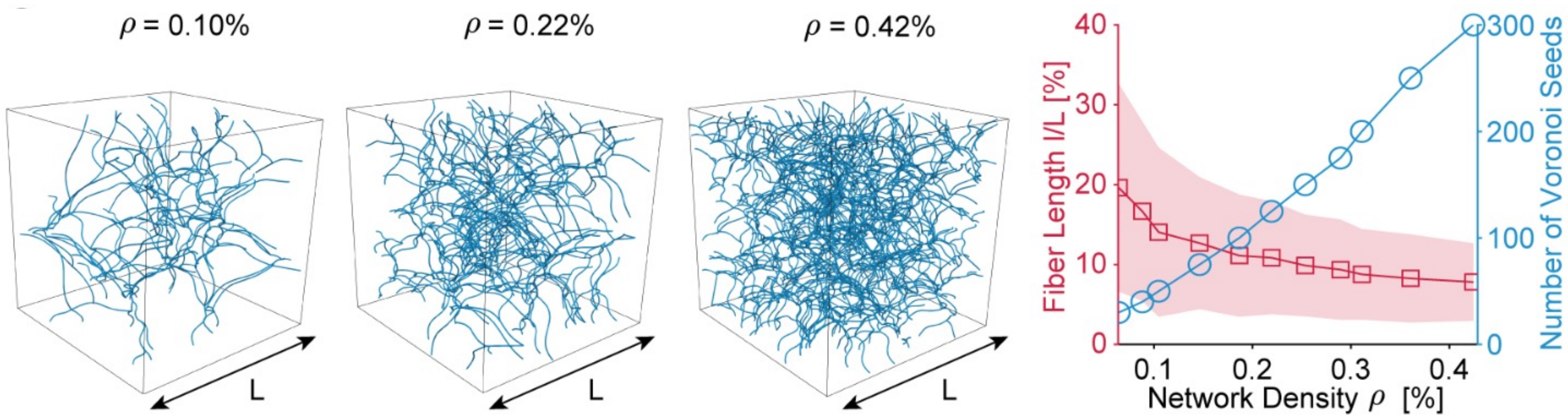


Size Effect

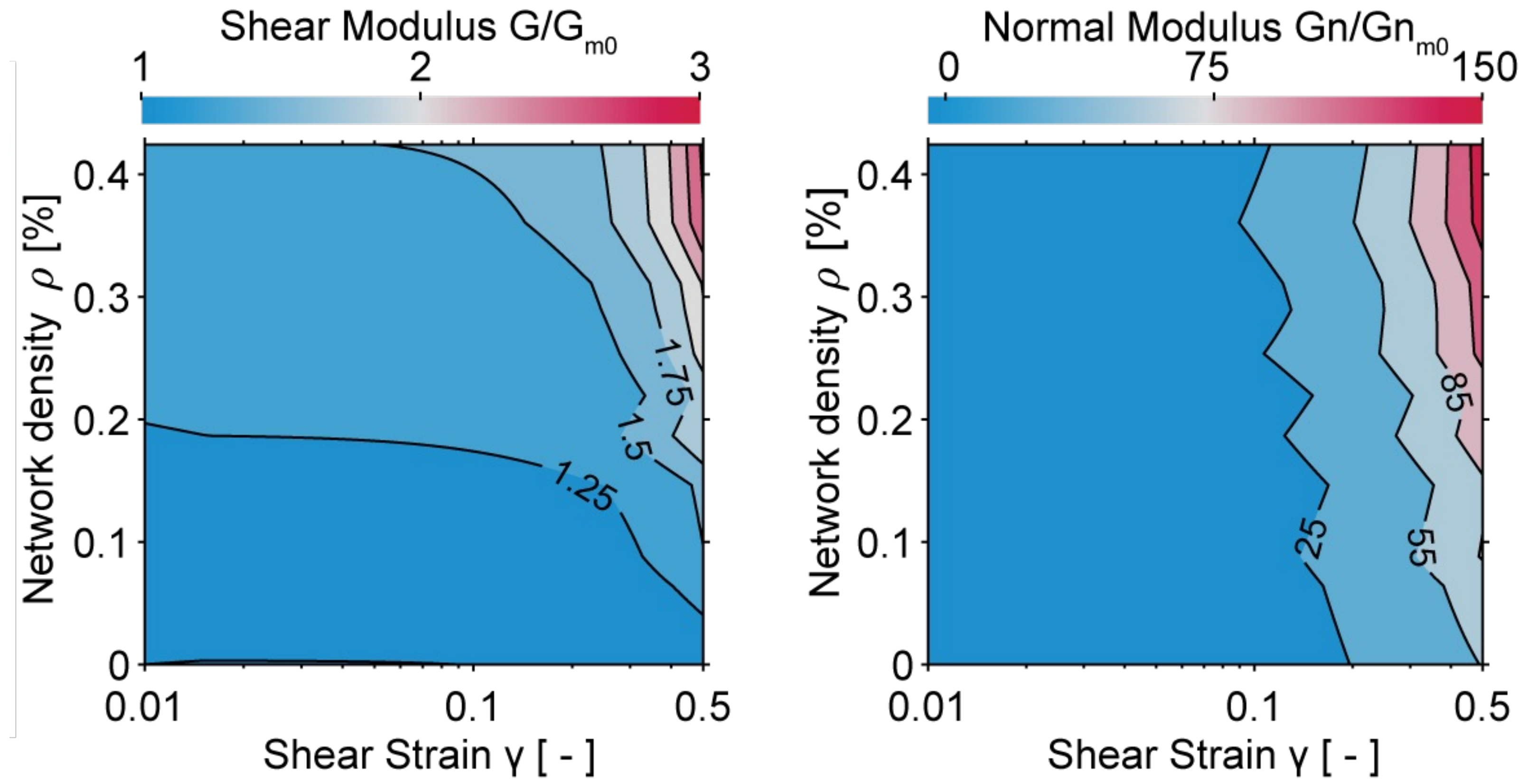
- Investigated the size effect on the apparent stiffness
  - Varying network sizes with same density
  - Shear Modulus
  - Normal Modulus
  - At three different fiber-to-matrix stiffness ratio Ef/Em
- The size effect is more prominent at
  - Higher fiber-to-matrix stiffness ratio
  - Shear modulus at the low-strain regime
- Highest density network converges sufficiently for both Shear and Normal moduli
  - Increasing fiber number the change in effective moduli decreases



Density Effect

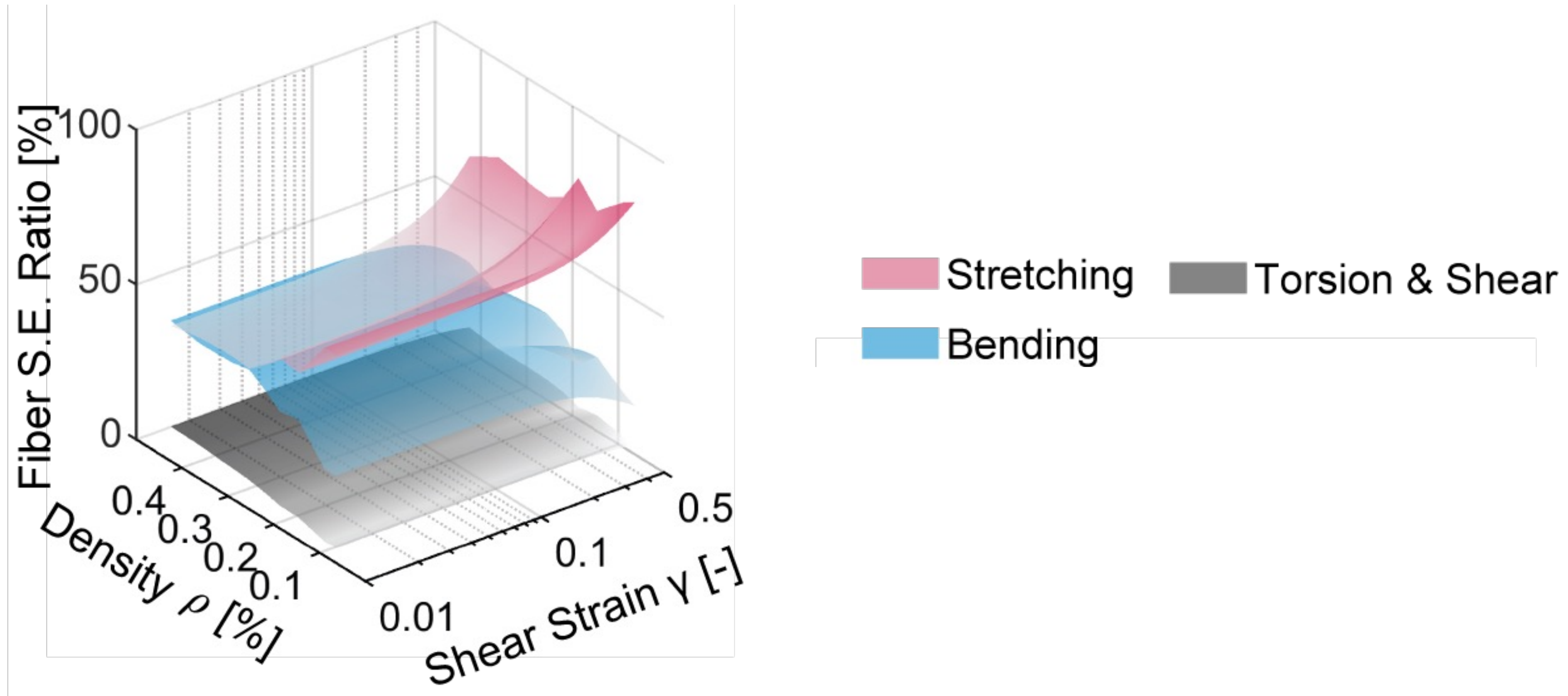


- Network Density Study
  - Constant network size (edge length)
  - Decreasing mean fiber length
  - Increasing network density
  - Simple Shear deformation



- Embedding fiber networks leads to
  - Strain stiffening behavior
  - A more pronounced negative Poynting effect

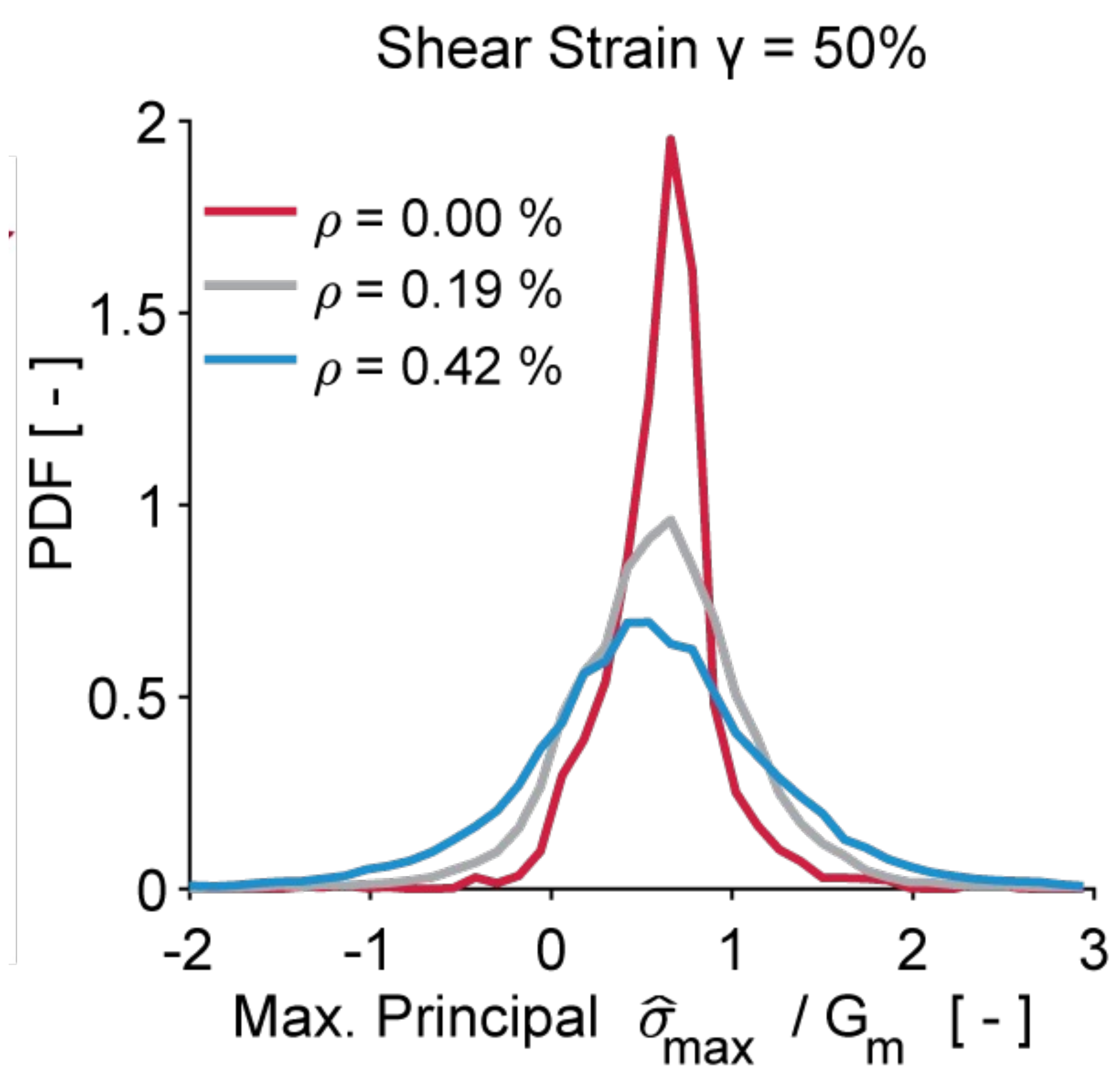
- From a strain energy perspective, these phenomena are driven primarily by fiber stretching, rather than bending or torsion

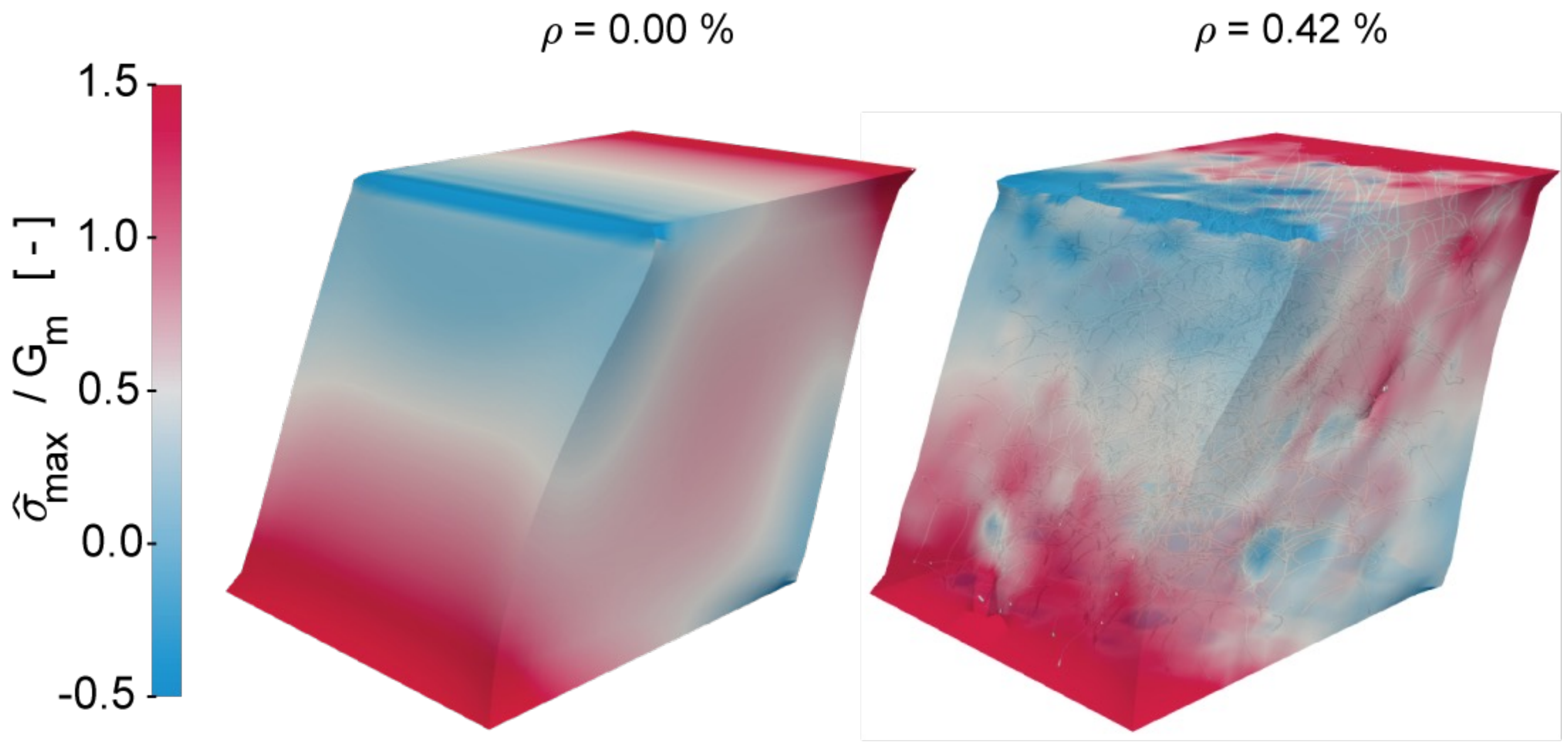


Networks and Matrix-stress Distribution

- Distribution of max. principal stress as a function of network density

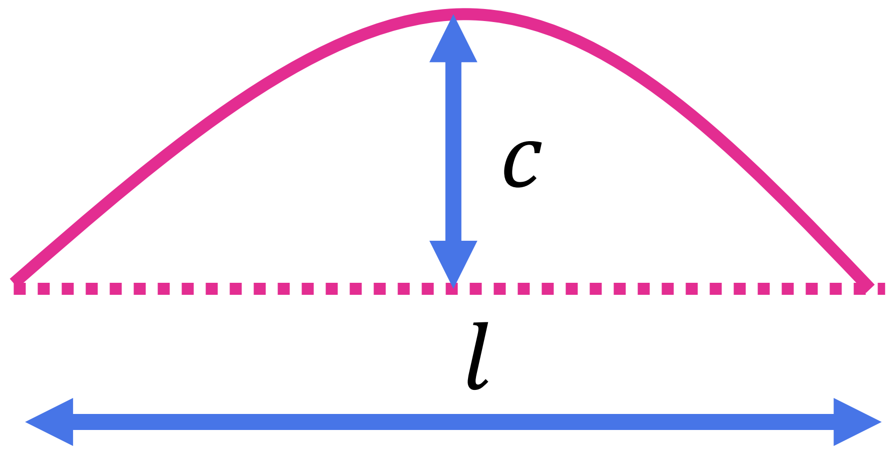
- Embedding networks introduce
  - Stress heterogeneity
  - Local stress concentrations





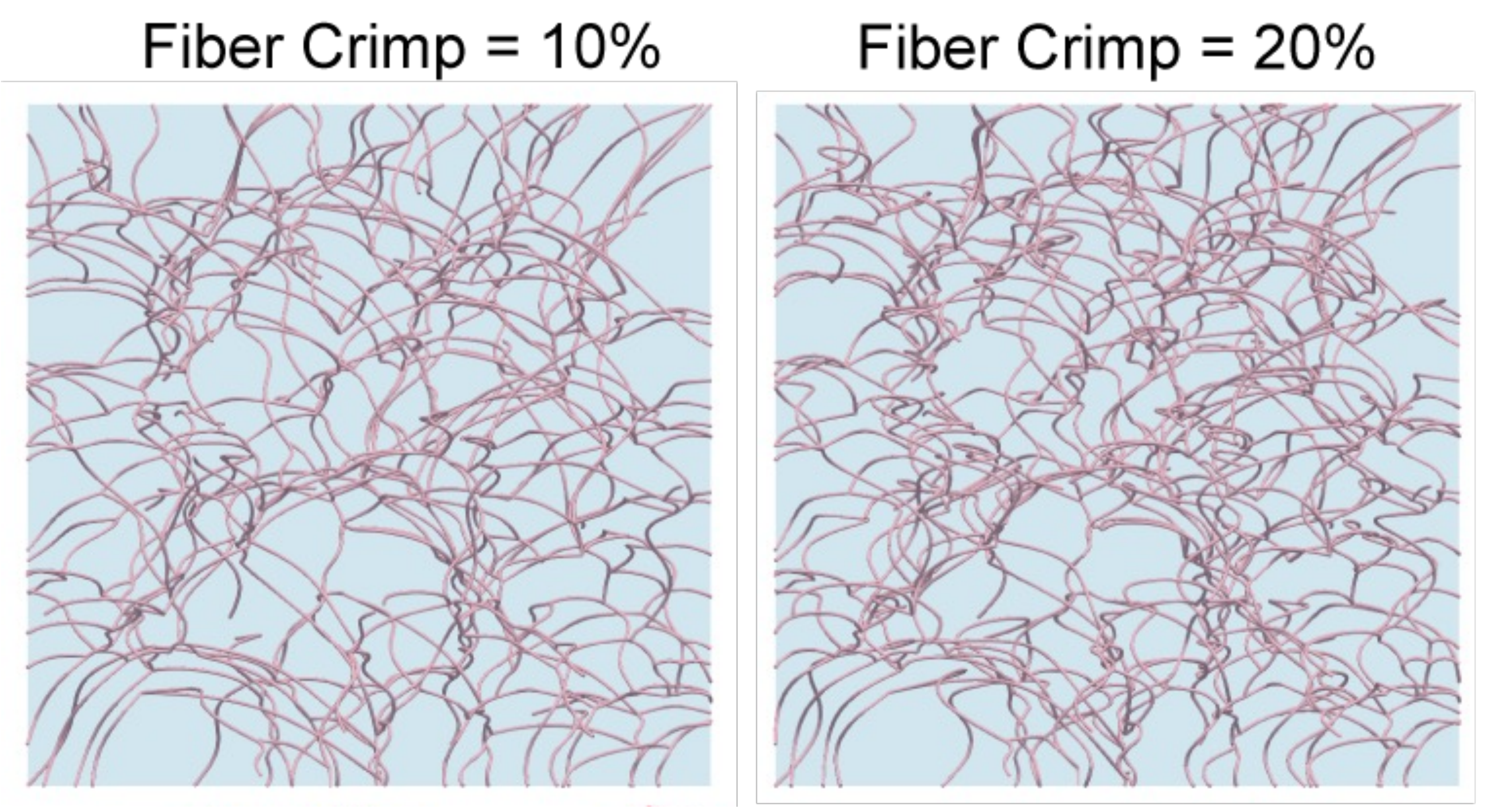
Fiber Strain Energy

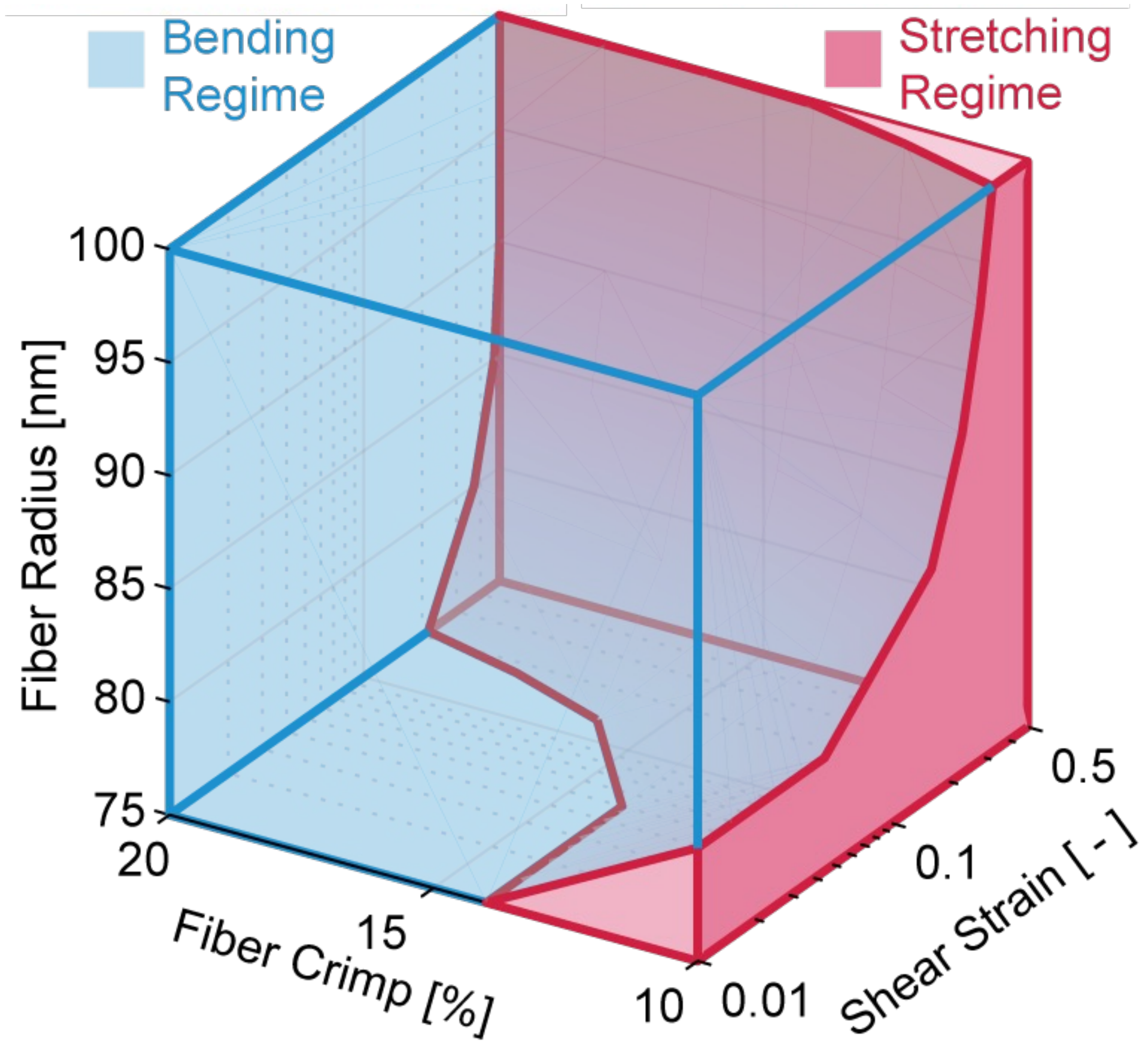
- Fiber crimp c/l [%] & fiber radius



- Stretching energy dominates for
  - Large deformations
  - Fibers with small undulations
  - Fibers with small radii

- Bending energy dominates for
  - linear/small deformations
  - fibers with large undulations
  - fibers with large radii





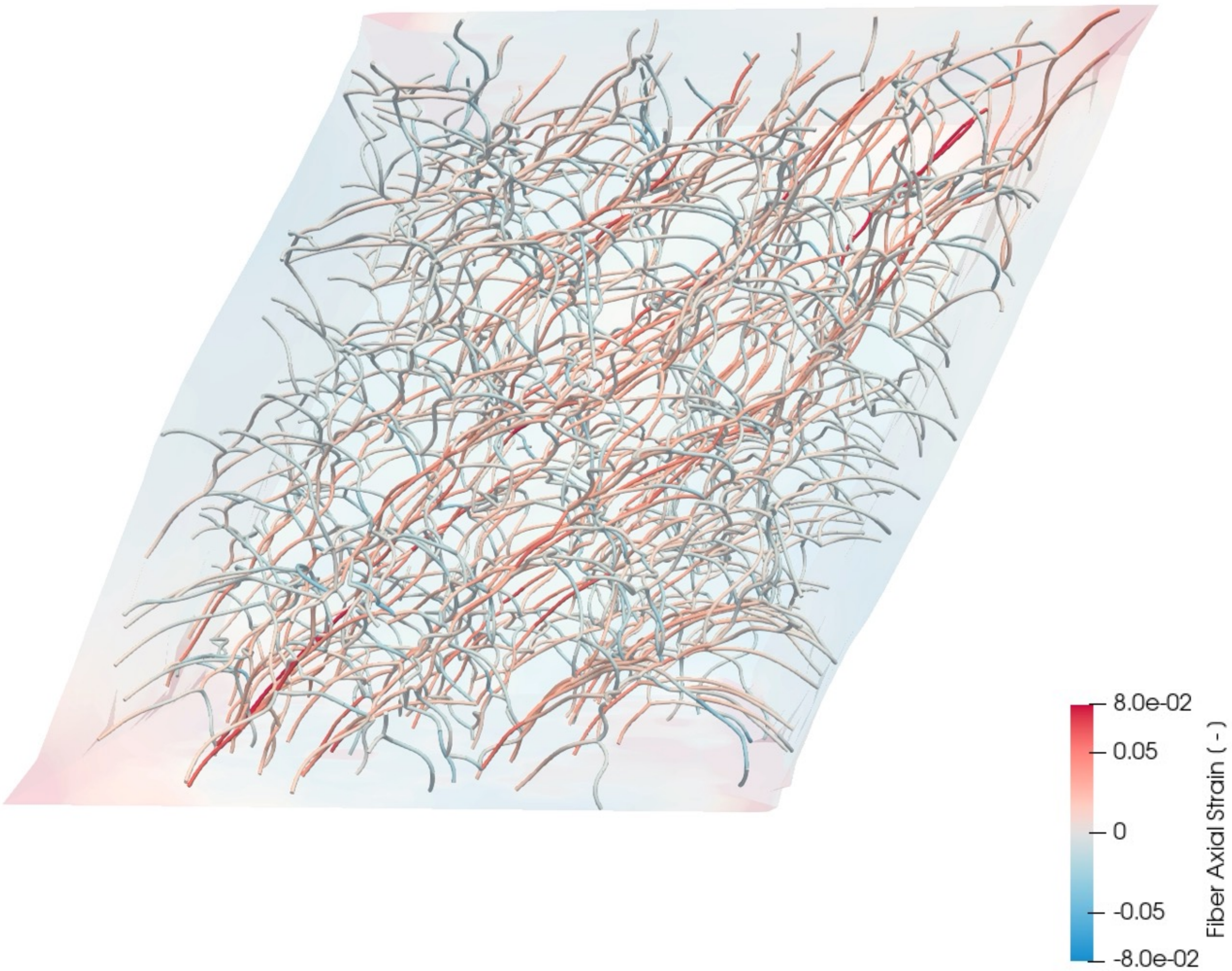
Conclusion

- Given the limitations presented (penalty parameter, mesh size, element length ratio), the mortar-type finite element approach can provide efficient models for embedded fiber networks.

- Embedding fiber networks leads to
  - Strain stiffening behavior
  - Negative Poynting effect
  - Stress heterogeneity

- Stretching (membrane) strain energy dominates the mechanics at large deformations.

- Future work
  - Interpret experimental data (blood clot modeling).
  - Expand on viscoelastic and/or damage-failure models of the fibers.



THANK YOU.

Dr. Manuel Rausch

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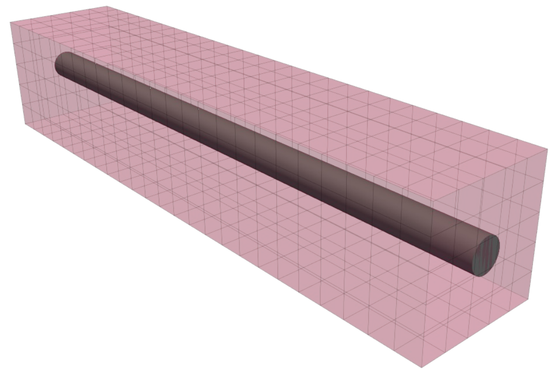
Theory



- Discretization of the solid, beam displacement and the Lagrange multiplier (LM) fields:

$$\underline{\boldsymbol{u}}_h{}^S = \sum_{k=1}^{n_S} N_k(\xi,\zeta,\eta)\boldsymbol{d}_k^S \qquad \underline{\boldsymbol{u}}_h{}^B = \sum_{l=1}^{n_B} H_l(\xi^B)\boldsymbol{I}^{3x3}\boldsymbol{d}_l^B \qquad \underline{\boldsymbol{\lambda}}_h = \sum_{l=1}^{n_\lambda} \Phi_j(\xi^B)\boldsymbol{\lambda}_j$$

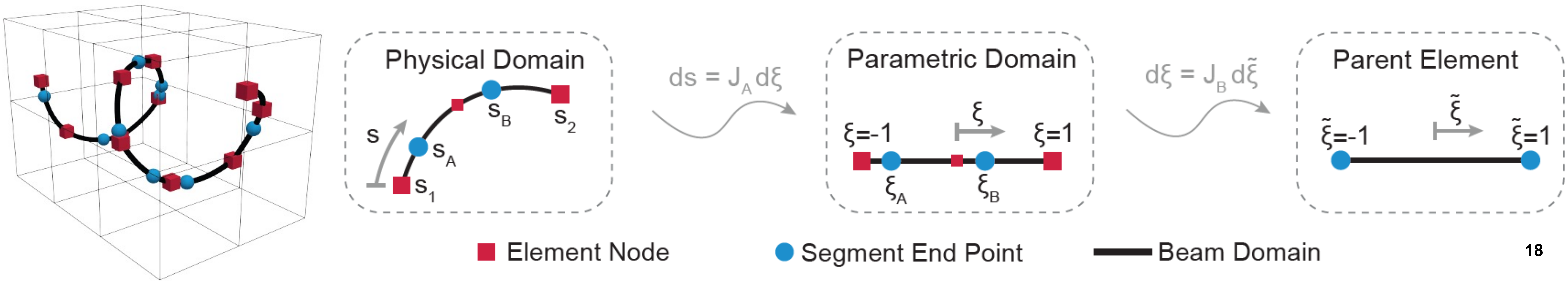
- Coupling matrices from δW^C (integration on the beam centerline):

$$\boldsymbol{D}^{(j,l)} = \int_{\Gamma^B_{c,h}} \Phi_j H_l(\xi^B) ds\, \boldsymbol{I}^{3x3} \qquad \boldsymbol{M}^{(j,k)} = \int_{\Gamma^B_{c,h}} \Phi_j N_k ds\ \ \boldsymbol{I}^{3x3} \qquad \boldsymbol{\kappa}^{(j,j)} = \int_{\Gamma^B_{c,h}} \Phi_j ds\, \boldsymbol{I}^{3x3}$$

LM node j ↔ Beam node l.

LM node j ↔ Solid node k
Projection to solid domain to evaluate N_k

Scaling matrix: LM node j



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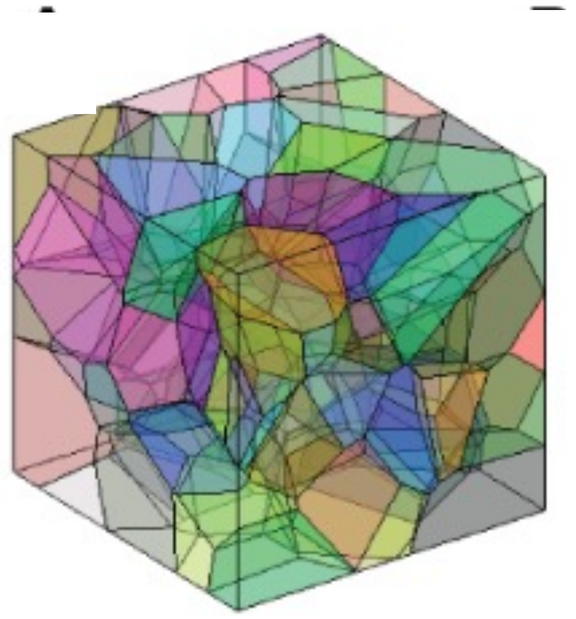
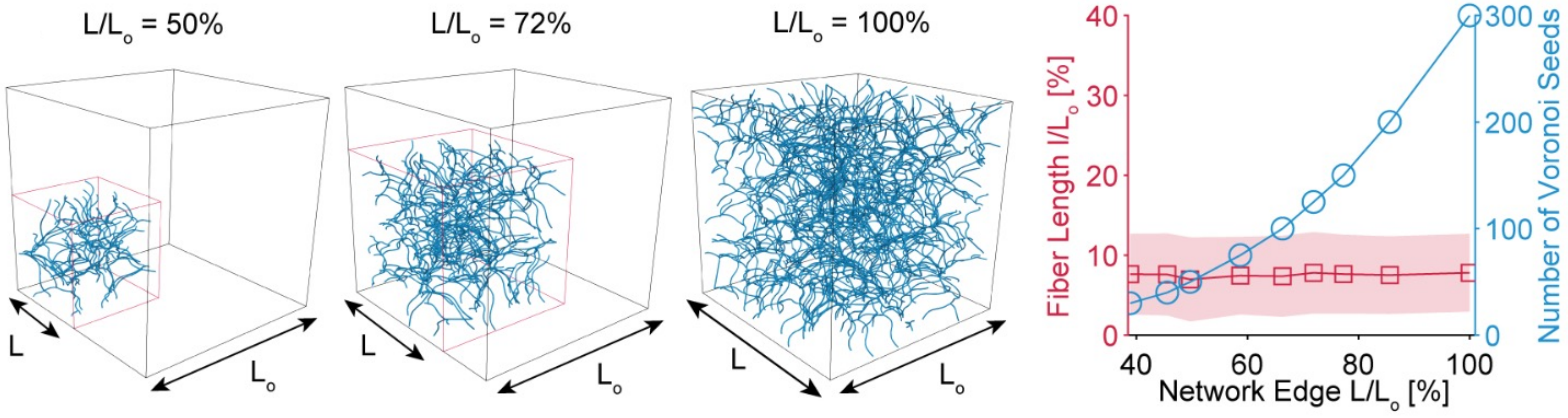


- Size Effect Study



- Density Effect study